Cambridge Assessment Admissions Testing

Test of Mathematics for University Admission

Paper 1 specimen paper hand-written worked answers

Cambridge Assessment Admissions Testing

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

Model Answers

PAPER 1

SPECIMEN

Time: 75 minutes

Additional Materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators must **NOT** be used. There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page

This question paper consists of 12 printed pages and 4 blank pages

BLANK PAGE

$x - 3y + 1 = 0$
$x = 3y - 1$

1. The sum of the two values of x that satisfy the simultaneous equations

$x - 3y + 1 = 0$ and $3x^2 - 7xy = 5$ is

$3(3y-1)^2 - 7(3y-1)y = 5$
$3(9y^2 - 6y + 1) - 21y^2 + 7y = 5$
$27y^2 - 18y + 3 - 21y^2 + 7y = 5$
$6y^2 - 11y - 2 = 0$

A −8.5

B −7.5

C −1.5

$\boxed{\text{D} \quad 3.5}$

E 4.5

F 5

$y = \frac{11 \pm \sqrt{121 + 4 \times 6 \times 2}}{12} = \frac{11 \pm \sqrt{169}}{12} = \frac{11 \pm 13}{12} = 2$ or $-1/6$

For $y = 2$, $x = 3 \times 2 - 1 = 5$

$y = -1/6$, $x = 3 \times -1/6 - 1 = -3/2$

Sum $= 5 + -3/2 = 3.5$

2. The number of solutions in the interval $0 \le \theta \le 4\pi$ of the equation

$\sin^2\theta + 3\cos\theta = 3$ is

$\sin^2\theta + \cos^2\theta = 1$ so...

$1 - \cos^2\theta + 3\cos\theta = 3$
$\cos^2\theta - 3\cos\theta + 2 = 0$

A 0

B 1

C 2

$\boxed{\text{D} \quad 3}$

E 4

F 5

G 6

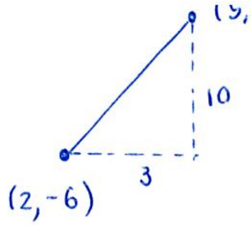
Let $y = \cos\theta$, then $y^2 - 3y + 2 = 0$
$(y-2)(y-1) = 0$

$\therefore$ $y = 2$ or 1 so $\cos\theta = 2$ or $\cos\theta = 1$
no real sol^ns $\theta = 0$

$\therefore$ $\theta = 0$ or 2π or 4π

3 solutions

[Turn over



Gradient $= \frac{10}{3}$ $\therefore$ gradient of perp. bisector $= \frac{-3}{10}$

3. The perpendicular bisector of the line segment joining the points $(2, -6)$ and $(5, 4)$ cuts the x-axis at the point with x-coordinate

common point is midpoint of segment: $(3.5, -1)$

A $\frac{1}{20}$

$\boxed{\text{B} \quad \frac{1}{6}}$

C $\frac{1}{3}$

D $\frac{19}{5}$

E $\frac{41}{6}$

Eqn of bisector is of form $y = mx + c$

ie. $y = \frac{-3}{10}x + c$

use $(3.5, -1)$ and get $-1 = \frac{-3}{10} \times 3.5 + c$

$-1 = \frac{-21}{20} + c \Rightarrow c = \frac{1}{20}$

so $y = \frac{-3}{10}x + \frac{1}{20}$

cuts x axis when $y = 0$, ie. $\frac{3}{10}x = \frac{1}{20} \Rightarrow x = \frac{1}{6}$

4. The complete set of values of x for which $(x^2 - 1)(x - 2) > 0$ is

$(x-1)(x+1)(x-2) > 0$

A $x < -1,\ 1 < x < 2$

B $x < -1,\ x > 2$

C $-1 < x < 2$

D $x < 1,\ x > 2$

$\boxed{\text{E} \quad -1 < x < 1,\ x > 2}$

Condition on x	$x-1$	$x+1$	$x-2$	$(x^2-1)(x-2)$
$x < -1$	−	−	−	−
$x = -1$	−	0	−	0
$-1 < x < 1$	−	+	−	+
$x = 1$	0	+	−	0
$1 < x < 2$	+	+	−	−
$x = 2$	+	+	0	0
$x > 2$	+	+	+	+

5. Given that $y = -\log_{10}(1-x)$ for $x < 1$, find x in terms of y.

$y = -\log_{10}(1-x)$

$-y = \log_{10}(1-x)$

$10^{-y} = 1-x$

$x = 1-10^{-y}$

A $x = -\frac{1}{\log_{10}(1-y)}$

B $x = 1 + \log_{10} y$

C $x = 1 - \log_{10} y$

$\boxed{\text{D} \quad x = 1 - 10^{-y}}$

E $x = 10^{-y} - 1$

F $x = 10^{1-y}$

6. It is given that $x+2$ is a factor of $x^3 + 4cx^2 + x(c+1)^2 - 6$.

The sum of the possible values of c is

$f(x) = x^3 + 4cx^2 + x(c+1)^2 - 6$

$(x+2)$ is a factor so $f(-2) = 0$

$f(-2) = (-2)^3 + 4c(-2)^2 + (-2)(c+1)^2 - 6$

$= -8 + 16c - 2(c+1)^2 - 6$

$= -14 + 16c - 2c^2 - 4c - 2$

$= -2c^2 + 12c - 16$

so $2c^2 - 12c + 16 = 0$

$c^2 - 6c + 8 = 0$

$(c-4)(c-2) = 0$

$\therefore c = 4$ or 2

$4 + 2 = 6$

A -10

B -6

C 0

$\boxed{\text{D} \quad 6}$

E 10

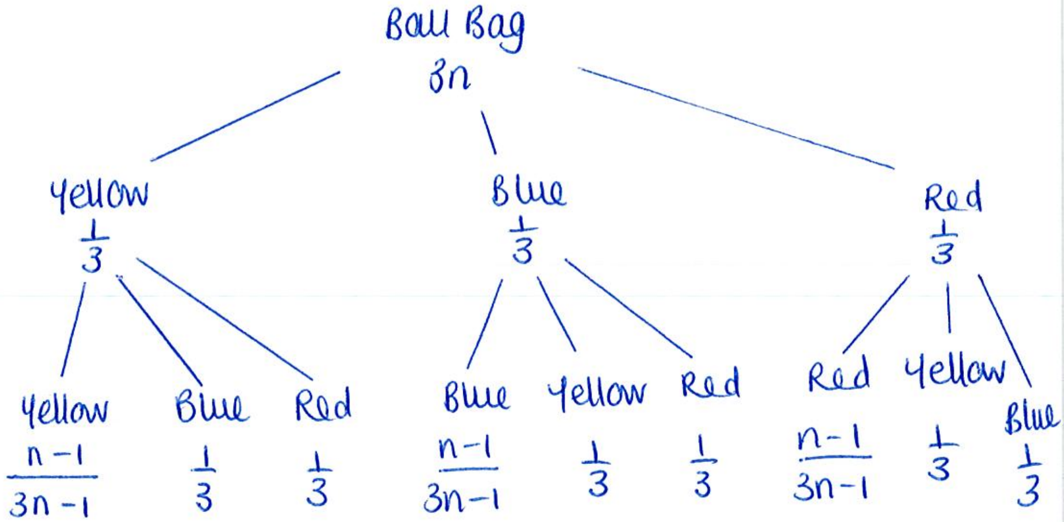
7. A bag contains n red balls, n yellow balls, and n blue balls. 3n balls

One ball is selected at random and not replaced.

A second ball is then selected at random and not replaced.

Each ball is equally likely to be chosen.

The probability that the two balls are **not** the same colour is



A $\frac{n-1}{3n-1}$

B $\frac{2n-2}{3n-1}$

C $\boxed{\frac{2n}{3n-1}}$

D $\frac{(n-1)^3}{27(3n-1)^3}$

E $\frac{3(n-1)}{3n-1}$

F $\frac{n^3}{27(3n-1)^3}$

Probability (Red Red) $= P(BB) = P(YY) = \frac{1}{3} \times \frac{n-1}{3n-1} = \frac{n-1}{3(3n-1)}$

probability (different) $= 1 - \frac{3(n-1)}{3(3n-1)} = 1 - \frac{n-1}{3n-1} = \frac{3n-1-n+1}{3n-1} = \frac{2n}{3n-1}$

8. Given that $a^x b^{2x} c^{3x} = 2$, where a, b, and c are positive real numbers, then $x =$

$\log(a^x b^{2x} c^{3x}) = \log 2$

$\log a^x + \log b^{2x} + \log c^{3x} = \log 2$

$x \log a + 2x \log b + 3x \log c = \log 2$

$x(\log a + \log b^2 + \log c^3) = \log 2$

$x = \dfrac{\log 2}{\log a + \log b^2 + \log c^3}$

$= \dfrac{\log 2}{\log ab^2c^3}$

A $\log_{10}\left(\frac{2}{a+2b+3c}\right)$

B $\frac{\log_{10} 2}{\log_{10}(a+2b+3c)}$

C $\frac{2}{\log_{10}(a+2b+3c)}$

D $\frac{2}{a+2b+3c}$

E $\log_{10}\left(\frac{2}{ab^2c^3}\right)$

F $\boxed{\frac{\log_{10} 2}{\log_{10}(ab^2c^3)}}$

G $\frac{2}{\log_{10}(ab^2c^3)}$

H $\frac{2}{ab^2c^3}$

9. The roots of the equation $2x^2 - 11x + c = 0$ differ by 2. The value of c is

A $\boxed{\frac{105}{8}}$

B $\frac{113}{8}$

C $\frac{117}{8}$

D $\frac{119}{8}$

$x = \dfrac{11 \pm \sqrt{121 - 4 \times 2 \times c}}{4} = \dfrac{11 \pm \sqrt{121-8c}}{4}$

Difference is 2 so:

$\dfrac{11+\sqrt{121-8c}}{4} - \dfrac{11-\sqrt{121-8c}}{4} = 2$

$\dfrac{2\sqrt{121-8c}}{4} = 2$

$\sqrt{121-8c} = 4$

$121 - 8c = 16$

$8c = 105$

$c = \dfrac{105}{8}$

10. The curve $y = \cos x$ is reflected in the line $y = 1$ and the resulting curve is then translated by $\frac{\pi}{4}$ units in the positive x-direction. The equation of this new curve is

$y = \cos x$

$\downarrow$

$y = \cos x - 1 \rightarrow y = 1 - \cos x \rightarrow y = 1 - \cos x + 1$
$= 2 - \cos x$

$\downarrow$

$y = 2 - \cos\left(x - \frac{\pi}{4}\right)$

A $\quad y = 2 + \cos\left(x + \frac{\pi}{4}\right)$

B $\quad y = 2 + \cos\left(x - \frac{\pi}{4}\right)$

C $\quad y = 2 - \cos\left(x + \frac{\pi}{4}\right)$

$\boxed{\text{D} \quad y = 2 - \cos\left(x - \frac{\pi}{4}\right)}$

11. The sum of the roots of the equation $2^{2x} - 8 \times 2^x + 15 = 0$ is

$(2^x)^2 - 8 \times 2^x + 15 = 0$

Let $y = 2^x$ then we have $y^2 - 8y + 15 = 0$
$(y-5)(y-3) = 0$

so $y = 5$ or 3

when $2^x = 5$ we get $x \log 2 = \log 5$
$x = \frac{\log 5}{\log 2}$

$2^x = 3$ $\quad x = \frac{\log 3}{\log 2}$

$\frac{\log 5}{\log 2} + \frac{\log 3}{\log 2} = \frac{\log 5 + \log 3}{\log 2} = \frac{\log 15}{\log 2}$

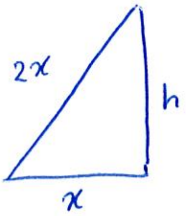
A $\quad 3$

B $\quad 8$

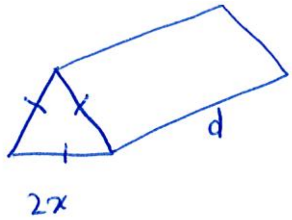
C $\quad 2\log_{10} 2$

D $\quad \log_{10}\left(\frac{15}{4}\right)$

$\boxed{\text{E} \quad \frac{\log_{10} 15}{\log_{10} 2}}$



$h = \sqrt{2^2 x^2 - x^2}$
$= \sqrt{4x^2 - x^2}$
$= \sqrt{3x^2}$
$= \sqrt{3}x$



12. The cross-section of a triangular prism is an equilateral triangle with side $2x$ cm. The length of the prism is d cm.

Let the total surface area of the prism be T cm². Given that the volume of the prism is T cm³, which one of the following is an expression for d in terms of x?

Area of triangle $= \frac{1}{2} \times 2x \times \sqrt{3}x = \sqrt{3}x^2$ $(\times 2)$

Area of rectangle $= 2xd$ $(\times 3)$

surface area $= 2\sqrt{3}x^2 + 6xd = T$

volume $= \sqrt{3}x^2 d = T$

A $\frac{x}{2x-3}$

B $\frac{3x}{3x-2\sqrt{3}}$

C $\frac{2x}{x-4\sqrt{3}}$

$\boxed{\text{D} \quad \frac{2x}{x-2\sqrt{3}}}$

E $\frac{2x}{x-\sqrt{3}}$

$2\sqrt{3}x^2 + 6xd = \sqrt{3}x^2 d$

$2\sqrt{3}x + 6d = \sqrt{3}xd$

$2\sqrt{3}x = \sqrt{3}xd - 6d = d(\sqrt{3}x - 6)$

$d = \frac{2\sqrt{3}x}{\sqrt{3}x-6} = \frac{2\sqrt{3}x}{\sqrt{3}x - 2\sqrt{3}\sqrt{3}} = \frac{2x}{x-2\sqrt{3}}$

13. How many real roots does the equation $x^4 - 4x^3 + 4x^2 - 10 = 0$ have?

$\frac{dy}{dx} = 4x^3 - 12x^2 + 8x = x(4x^2 - 12x + 8) = 4x(x^2 - 3x + 2) = 4x(x-2)(x-1)$

Stationary points at: $x = 0$ & $y = -10$
$x = 1$ $y = -9$
$x = 2$ $y = -10$

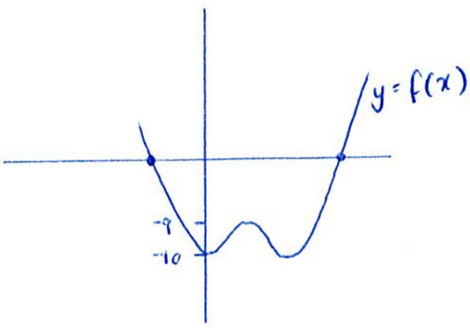
A 0

B 1

$\boxed{\text{C} \quad 2}$

D 3

E 4



$f(x) = 0$ in 2 places

[Turn over

Straight lines where $y = mx + c$ or in this case with logs:

Let $c = \log C$

$\log y = m \log x + c$

$\log y = \log x^m + c$

$\log y = \log x^m + \log C$

$\log y = \log Cx^m$

$y = Cx^m$

14. a, b, x, and y are real and positive.

a and b are constants.

x and y are related.

A graph of $\log y$ against $\log x$ is drawn.

For which one of the following relationships will this graph be a straight line?

A $y^b = a^x$

B $y = ab^x$

C $y^2 = a + x^b$

D $\boxed{y = ax^b}$

E $y^x = a^b$

15. The smallest possible value of $\int_0^1 (x-a)^2\, dx$ as a varies is

A $\boxed{\frac{1}{12}}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{7}{12}$

E 2

$$\int_0^1 (x-a)^2\, dx = \int_0^1 x^2 - 2ax + a^2\, dx$$

$$= \frac{x^3}{3} - \frac{2ax^2}{2} + a^2 x \Big|_0^1$$

$$= \frac{1}{3} - a + a^2$$

$$\frac{d}{da} = 2a - 1$$

$2a - 1 = 0$

$a = 1/2$

Sub back in: $\frac{1}{3} - a + a^2 = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{1}{12}$

16. Given that c and d are non-zero integers, the expression $\frac{10^{c-2d} \times 20^{2c+d}}{8^c \times 125^{c+d}}$ is an integer if

A $c < 0$

B $d < 0$

C $c < 0$ and $d < 0$

D $c < 0$ and $d > 0$

$\boxed{\text{E} \quad c > 0 \text{ and } d < 0}$

F $c > 0$ and $d > 0$

G $d > 0$

H $c > 0$

$$= \frac{(2 \times 5)^{c-2d} \times (2^2 \times 5)^{2c+d}}{(2^3)^c \times (5^3)^{c+d}}$$

$$= \frac{2^{c-2d+2(2c+d)} \times 5^{c-2d+2c+d}}{2^{3c} \times 5^{3(c+d)}}$$

$$= \frac{2^{5c} \times 5^{3c-d}}{2^{3c} \times 5^{3c+3d}}$$

$$= 2^{2c} \times 5^{-4d}$$

This is an integer when 2c & -4d are non negative integers

we're told c&d are non zero integers ∴ we need $c > 0$ & $d > 0$.

17. For what values of the non-zero real number a does the quadratic equation $ax^2 + (a-2)x = 2$ have real distinct roots?

A All values of a

B $a = -2$

C $a > -2$

$\boxed{\text{D} \quad a \neq -2}$

E No values of a

$$x = \frac{-(a-2) \pm \sqrt{(a-2)^2 + 4a \times 2}}{2a}$$

$$= \frac{2-a \pm \sqrt{(a-2)^2 + 8a}}{2a}$$

Real distinct roots when $\sqrt{(a-2)^2 + 8a} > 0$

$(a-2)^2 + 8a = a^2 - 4a + 4 + 8a = a^2 + 4a + 4 = (a+2)^2$

Need $(a+2)^2 > 0$ which happens $\forall a$ except $a = -2$

18. The angle x is measured in radians and is such that $0 \le x \le \pi$.

The total length of any intervals for which $-1 \le \tan x \le 1$ and $\sin 2x \ge 0.5$ is

A $\frac{\pi}{12}$

$\boxed{\text{B} \quad \frac{\pi}{6}}$

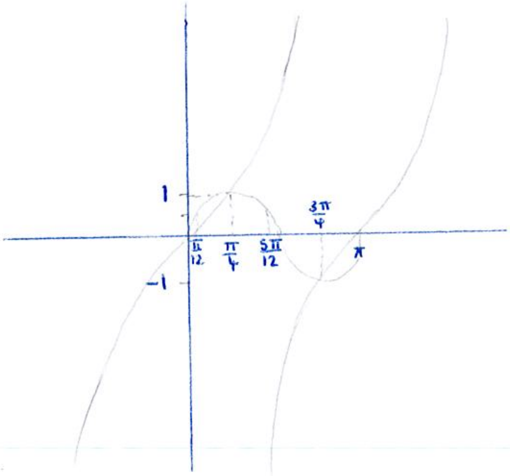
C $\frac{\pi}{4}$

D $\frac{\pi}{3}$

E $\frac{5\pi}{12}$

F $\frac{\pi}{2}$

G $\frac{5\pi}{6}$



For $y = \tan x$

$-1 \le \tan x \le 1$

when $0 \le x \le \frac{\pi}{4}$

$\frac{3}{4}\pi \le x \le \pi$

For $y = \sin 2x$

$y = \sin\frac{\pi}{6} = \sin\frac{\pi}{6} = 0.5$

$\sin 2x \geq 0.5$

when $\frac{\pi}{6} \le 2x \le \frac{5\pi}{6}$

$\frac{\pi}{12} \le x \le \frac{5}{12}\pi$

Intervals all satisfied for

$\frac{\pi}{12} \le x \le \frac{\pi}{4}$

$\frac{\pi}{4} - \frac{\pi}{12} = \frac{\pi}{6}$

19. A geometric series has first term 4 and common ratio r, where $0 < r < 1$.

$4, 4r, 4r^2, 4r^3, \ldots$

The first, second, and fourth terms of this geometric series form three successive terms of an arithmetic series. $4, 4+d, 4+2d, 4+3d, \ldots$

The sum to infinity of the geometric series is

A $\frac{1}{2}(\sqrt{5} - 1)$

B $2(3 - \sqrt{5})$

C $2(1 + \sqrt{5})$

$\boxed{\text{D} \quad 2(3 + \sqrt{5})}$

$4r = 4 + d$

$d = 4r - 1$

$= 4(r-1) \quad (1)$

$4r^3 = 4 + 2d$

$2d = 4r^3 - 4 \quad (2)$

(2) − (1) gives $d = 4r^3 - 4 - 4r + 4 = 4r^3 - 4r$

$= 4r(r^2 - 1)$

$= 4r(r-1)(r+1)$

$4(r-1) = 4r(r-1)(r+1) \Rightarrow r(r+1) = 1$

$r^2 + r - 1 = 0$

$r = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$ we're told $r > 0$ so $r = \frac{-1+\sqrt{5}}{2}$

$S_\infty = \frac{a}{1-r} = \frac{4}{1 - \frac{-1+\sqrt{5}}{2}} = \frac{4}{\frac{3-\sqrt{5}}{2}} = \frac{8}{3-\sqrt{5}} = \frac{8(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = 6 + 2\sqrt{5} = 2(3+\sqrt{5})$

20. The coefficient of x^2 in the expansion of $(4-x^2)[(1+2x+3x^2)^6-(1+4x^3)^5]$ is

A 28

B 72

C 78

D 192

E 240

F 310

$\boxed{\text{G} \quad 312}$

$$\begin{array}{ccccccccccccc} & & & & & & 1 & & & & & & \\ & & & & & 1 & & 1 & & & & & \\ & & & & 1 & & 2 & & 1 & & & & \\ & & & 1 & & 3 & & 3 & & 1 & & & \\ & & 1 & & 4 & & 6 & & 4 & & 1 & & \\ & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \text{5th} \\ 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \quad \text{6th} \end{array}$$

END OF TEST

$(1+2x+3x^2)^6$

$= 1^6 + 6(1)^5(2x+3x^2) + 15(1)^4(2x+3x^2)^2 + \dots$

$= 1 + 6(2x+3x^2) + 15(2x+3x^2)(2x+3x^2) + \dots$

$= 1 + 12x + 18x^2 + 15(4x^2+12x^3+9x^4) + \dots$

$= 1 + 12x + 18x^2 + 60x^2 + 180x^3 + 135x^4 + \dots$

$= 1 + 12x + 78x^2 + \dots$

$(1+4x^3)^5$

$= 1 + \dots$

$(4-x^2)(1+12x+78x^2+\dots-1+\dots)$

$= 4 + \dots + (4\times 78)x^2 + \dots - x^2 + \dots + x^2 + \dots$

coefficient is $4 \times 78 = 312$

[Turn over

BLANK PAGE

BLANK PAGE

BLANK PAGE

We are Cambridge Assessment Admissions Testing, part of the University of Cambridge. Our research-based tests provide a fair measure of skills and aptitude to help you make informed decisions. As a trusted partner, we work closely with universities, governments and employers to enhance their selection processes.

Cambridge Assessment
Admissions Testing
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA
United Kingdom

Admissions tests support:
admissionstesting.org/help